

5. PROPORTIONAL EFFECT.

5.1. Introduction.

It happens often in practise that the structural functions of different data sets of data collected in the same neighbourhood, exhibit some differences. These are frequently due to the unstationary behaviour of the measured parameter. When having sufficient data, one can apply the concept of quasi stationarity, and eliminate the differences by applying scaling of the data. In case of observing similarities of the data sets after having applied scaling of the data, a proportional effect is said to be observed.

5.2. Quasi stationarity.

The concept of quasi stationarity is necessary before any further discussion of the concept of the proportional effect. Suppose a structural function (covariance or variogram), is only used for a limited distance ($|h| < b$). The limit b , can represent the extent of a homogeneous zone with respect to the parameter under study, or the region which can be considered as neighbourhood. The concept of stationarity can be applied only within this limit. The hypothesis of quasi stationarity, assumes that;

1. The expectation of the parameter is quasi constant

over the limited neighbourhood. For a pair of points x and x' belonging to the same neighbourhood $V(x_0)$ centered on the point x_0 , it is

$$m(x) = m(x') = m(x_0) \quad (5.1)$$

2. Inside such a neighbourhood $V(x_0)$, the structural functions γ or C , depend only on the vector of the separating distance h , and not on the two positions x and x' . Evidently, they also depend on the particular neighbourhood $V(x_0)$, that is, on the point x_0 .

$$\gamma(x, x') = \gamma(x - x', x_0) \quad \forall x, x' \in V(x_0) \quad (5.2)$$

5.3. Proportional effect.

The proportional effect is just an experimental observation. It can be interpreted by the fact that the random function is only locally and quasi stationary. It supposes that the experimental structural functions such as the experimental variograms $\gamma(h, x_0)$, $\gamma(h, x'_0)$, ... on data sets of different neighbourhoods, can be made to coincide by dividing each one of them by a function of the corresponding experimental mean of the available data set in each neighbourhood:

$$\gamma(h, x) / f(m^*(x)) = \gamma(h, x') / f(m^*(x'_0)) \quad (5.3)$$

where $m^*(x_0)$ is the experimental mean.

This amounts to the assumption of the existence of a stationary model of the structural function (in the case of the

variogram $\gamma_o(h)$, independent of the neighbourhood location x_o , and such that :

$$\gamma(h, x_o) = f(m^*(x_o)) \gamma_o(h)$$

In such case, the variograms $\gamma(h, x_o)$ and $\gamma(h, x'_o)$, are said to differ from one another by a proportional effect.

The function f of the proportional effect, can be determined separately in each case, by studying the proportional relationship between various experimental variograms, coming from different neighbourhoods. It is evident that sufficient amounts of data are required for each neighbourhood.

5.3.1. Direct and inverse proportionality.

In cases where the experimental structural function (variogram) increases with the corresponding experimental mean, the effect is said to be direct. It normally occurs when the random variable has a lognormal type histogram, i.e. the mode is less than the expectation.

When, on the other hand, the experimental variogram decreases with the increase of the experimental mean, the proportional effect is said to be inverse. This occurs when the random variable has an inverse lognormal type histogram, with the mode greater than the expectation.

5.3.2. The proportional relationship.

The proportional function f , can be a relationship of any form with the experimental mean:

$$f(m^*(x)) = m^*(x) \quad (5.4)$$

$$\text{or} \quad = (m^*(x))^2 \quad (5.5)$$

$$\text{or} \quad = (A - m^*(x))^2 \quad (5.6)$$

Both equations 5.5 and 5.6, represent an effect with a relation which is a function of $(m^*(x))^2$. In the first case though, we have direct, and in the second, inverse proportionality. It can also be a non-linear relation.

It has been suggested that when the logarithm of the parameter exhibits intrinsic features, the relation can be expressed in terms of its experimental means, without taking the mean from the logarithmic transformation (David, 1977).

5.4. Study of the proportional effect.

The results obtained from the present study, have been compared to the two previous ones (table 8, chapter 3). No resemblance was found with none of them, neither in the K , nor in the $\log K$ data sets. It can be concluded that the data come from different areas.

In the second study (Tan, 1986), a proportional effect investigation gave positive results. An inverse proportional relation was found in the variograms of the log of K , having a

function f equal simply to the mean of the K values.

The same proportional relation was investigated in the present study. In figure the variogram of the $\log K$ values of the present study was plotted, together with the variograms of the two previous studies, multiplied by the ratios of the means of the K values (106.9 for the first study and 1628.4 for the second one). It is shown that the points of the first study lie roughly on a straight line with the ones of the present one. On the other hand, the points of the second study, lie much higher. So, there is a direct proportional relationship between the present study and the study of the 14x14m plot, while with the other one, no such effect is detected. Since there is an inverse proportional relationship between the two previous studies, this should be expected.

Table 19. Comparison of the functional structures with previous studies.

Data set :		Nurul	Tan	Present	Pre./Nur.	Pre./Tan
mean	1	0.6155	0.0404	65.788	106.9	1628.4
	2	-0.2451	-2.0070	0.491	-2.0	-0.2
variance	1	0.0688	0.0089	10090.000	146721.0	1133707.9
	2	0.0289	0.6636	3.062	106.1	4.6
mean /var.	1	5.5088	0.1834	0.429	0.078	2.3
nugget	2	0.0266	0.4157	0.000	0.	0.0
slope	2	0.0002	0.0030	0.027	147.5	9.0

Note : 1 stands for K values, 2 for $\log K$ values

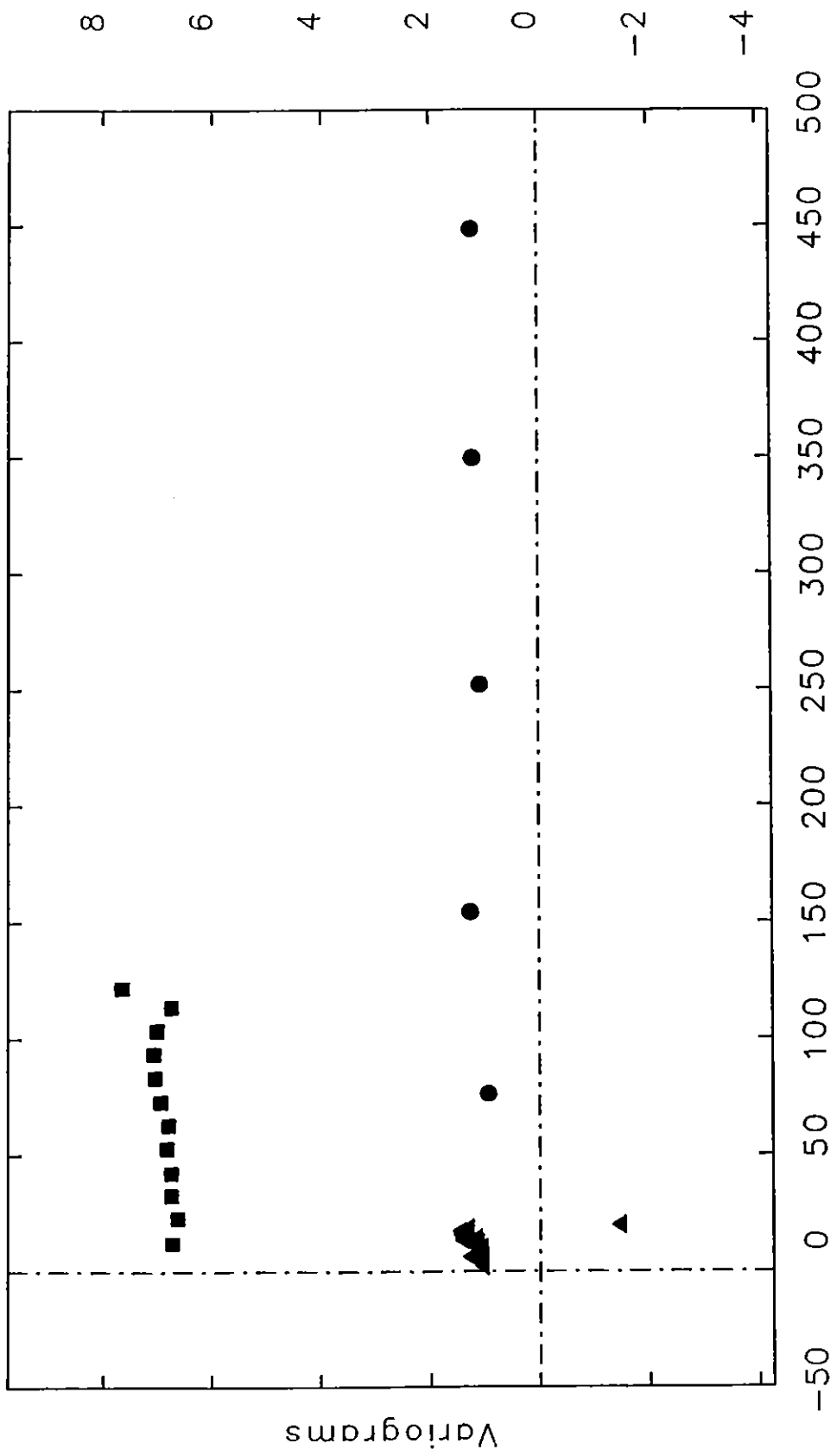


fig. 30. Variogram of log of K after applying proportionality

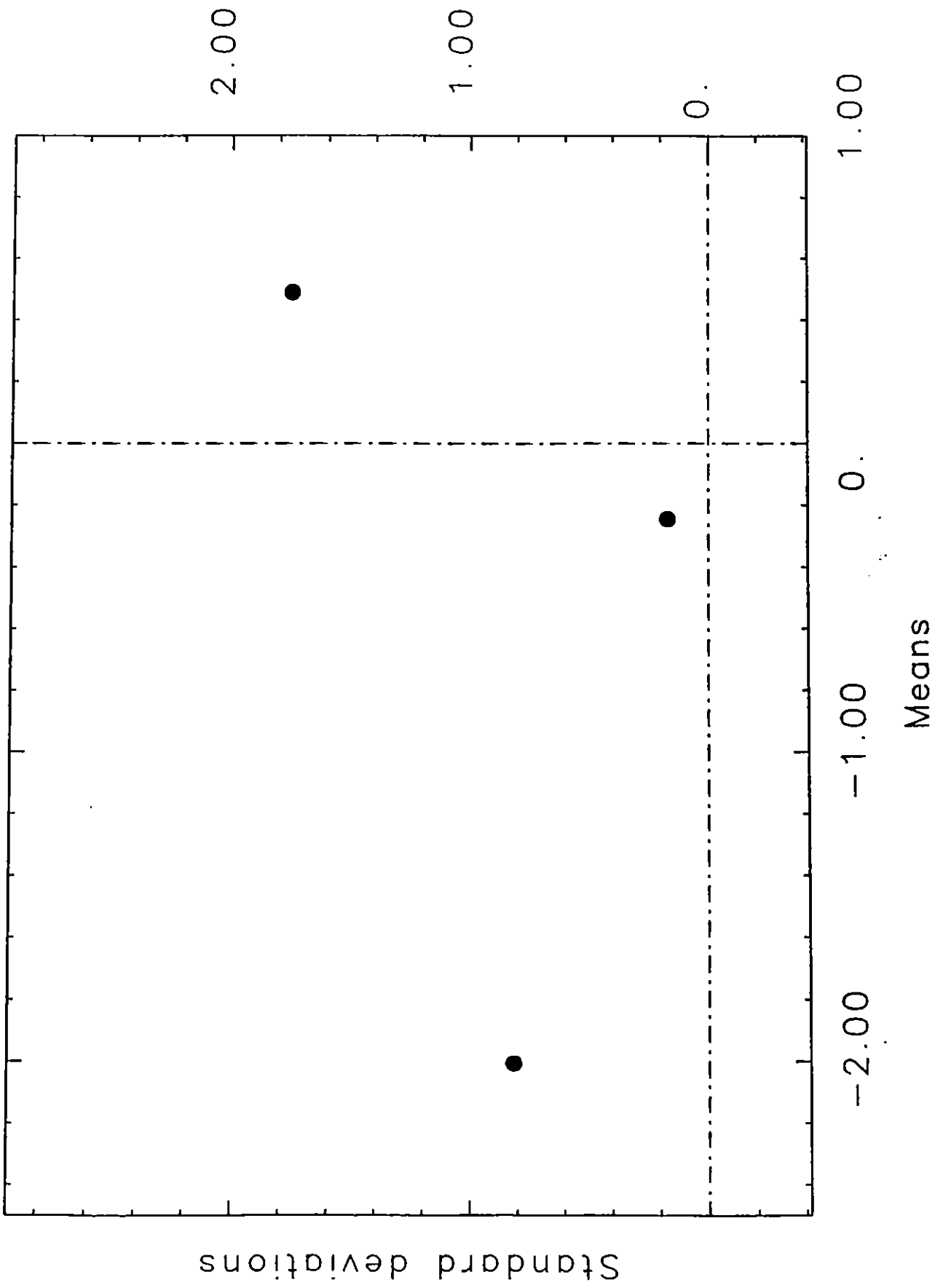


fig. 31. Standard deviations versus means

6. KRIGING

6.1. Introduction

Kriging is a method used for the estimation of the value of a field parameter and its variance. It is applied when the parameter can be considered as a regionalised variable. It was introduced by Matheron (1960) for the evaluation of mineral resources. The estimation can be local or global. Local estimation, refers to point estimation, and the global estimation extends to averages over a certain area.

6.2. Kriging

As Kriging, is defined an estimation technique which provides a best linear unbiased estimator of a regionalised variable (usually referred to as B.L.U.E.). It is assumed that the intrinsic hypothesis still holds.

Considering a point x having the value of an unknown parameter $Z(x)$, and a series of n observations $Z(x_1), \dots, Z(x_n)$ at x_1, \dots, x_n respectively, a set of weight coefficients a_1, \dots, a_n is chosen, which will make the weighted average Z^* the best estimator.

$$Z_o^* = \sum_i a_i Z(x_i) \quad (6.1)$$

For an optimal, we impose two conditions:

1. It must be unbiased.

$$E(Z_0^e) = E(Z_0) = m \quad (6.2a)$$

$$E(\sum_i a_i Z_i - Z_0) = 0 \quad (6.2b)$$

which implies that

$$\sum_i a_i = 1 \quad (6.2c)$$

2. The minimisation of the estimation variance.

The kriging estimation variance, is given by:

$$\begin{aligned} \sigma^2 &= E((Z_0^e - Z_0)^2) \\ &= E((\sum_i a_i Z_i - \sum_i a_i Z_0)^2) \\ &= \sum_{ij} a_i a_j E((Z_i - Z_0)(Z_j - Z_0)) \end{aligned} \quad (6.3)$$

The variogram, can be by definition written as:

$$\begin{aligned} \gamma_{ij} &= (1/2)E((Z_i - Z_j)^2) \\ &= (1/2)E((Z_i - Z_0)^2) + (1/2)E((Z_j - Z_0)^2) - E((Z_i - Z_0)(Z_j - Z_0)) \\ &= \gamma_{i0} + \gamma_{j0} - E((Z_i - Z_0)(Z_j - Z_0)) \end{aligned}$$

$$\text{or} \quad E((Z_i - Z_0)(Z_j - Z_0)) = \gamma_{i0} + \gamma_{j0} - \gamma_{ij} \quad (6.4)$$

By substituting the equation 6.4 in 6.3, we obtain:

$$= -\sum_i \sum_j a_i a_j \gamma_{ij} + \sum_i a_i \gamma_{i0} \quad (6.5)$$

By applying the principle of the Lagrange multiplier, we can minimise the sum in equation 6.5, under the condition of equation 6.2c. We then have to minimise:

$$F = Q + 2\mu c \quad (6.6)$$

where Q is the function in equation 6.5

μ is the Lagrange multiplier

c is zero when there is a constant and represents the condition.

By substituting from equation 6.5 and taking the partial

derivatives with respect to a and μ , a system of $n+1$ linear equations with $n+1$ unknowns is obtained. This system is usually referred to as the "kriging system" (equations 6.7 and 6.8):

$$\frac{\partial F}{\partial a_i} = -\sum_j \gamma_{ij} a_j + \gamma_{i0} - \mu = 0 \quad (6.7)$$

$$\frac{\partial F}{\partial \mu} = \sum_i a_i = 1 \quad (6.8)$$

The first of the two equations, can be expressed in matrix form as follows:

$$(\Sigma)(A) = (D) \quad (6.9)$$

$$\begin{bmatrix} \gamma_{11} & \gamma_{12} & \dots & \gamma_{1n} & 1 \\ \gamma_{21} & \gamma_{22} & \dots & \gamma_{2n} & 1 \\ \vdots & \vdots & & \vdots & \vdots \\ \gamma_{n1} & \gamma_{n2} & \dots & \gamma_{nn} & 1 \\ 1 & 1 & \dots & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \\ \mu \end{bmatrix} = \begin{bmatrix} \gamma_{10} \\ \gamma_{20} \\ \vdots \\ \gamma_{n0} \\ 1 \end{bmatrix}$$

(Σ) is a symmetric matrix depending only on the observations,
 (D) depends on both unknown and observation points.

Solving this system will result to the n coefficients a_i and the Lagrange multiplier. With the known values of a_i , we can compute the kriging estimator. The kriging variance, can be expressed in terms of the variogram:

$$\sigma_z^2 = E((Z_o^* - Z_o)^2) = \mu + \sum_i \gamma_{i0} a_i \quad (6.10)$$

6.3. Properties of Kriging

The kriging system considers the distance between the point under estimation and the data points, with "io" terms, the distances between data points with "ij" terms, and the structures of the variables through the variogram.

When kriging at x_0 which does not coincide with any observation point x_i , the kriging estimators in $x_0 \neq x_i$ give a smooth curve (fig. 32a). But when kriging is carried out at a point that tends towards one of the observation points x_i , the value of the estimator will change and give a sudden jump when it coincides with the observation point (fig. 32b). This is due to the presence of the nugget effect. The value obtained here, will be the same with the observed value, with a variance equal to zero. This is obvious, since kriging is done at a point which is the exact interpolator.

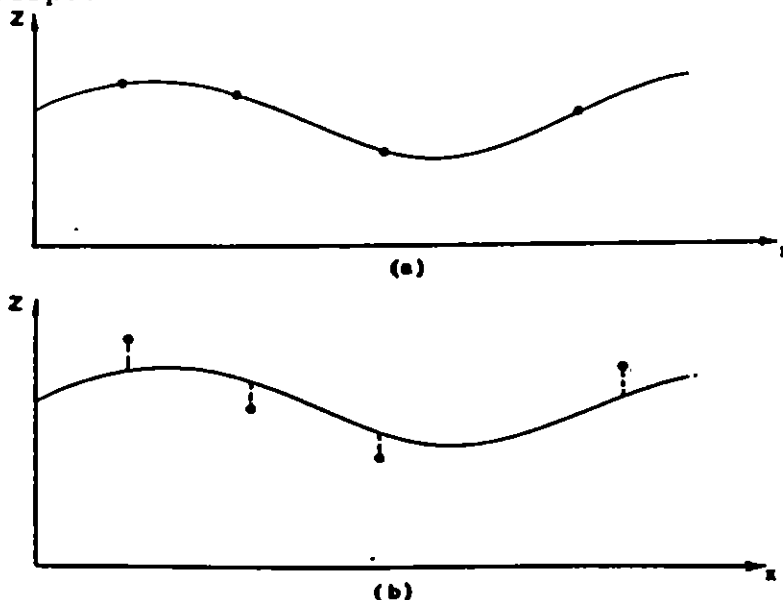


Fig. 32. Kriging position

6.4. Kriging with nugget filtering.

Kriging is carried out with the inclusion of some errors in the regionalised variable, which can be experimental, random, or microregionalisation errors. These are expressed in the nugget effect of the variogram. The position of the kriging cannot be on the observation point itself, since it will give a discontinuity. Kriging at a point beside an observational one, will give a best estimator.

It is possible to filter out the errors. They may consist of all possible kinds of error components, but here it is considered that the main ones are the measurement errors.

Suppose that the observations possess a certain type of error δ , not correlated with $Z(x)$, and having a mean $E(\delta)$ equal to zero and variance σ_δ^2 . Then the kriging estimator can be expressed as:

$$Z_o'' = \sum a_i' (Z_i + \delta_i) \quad (6.11)$$

The expectation of the estimator:

$$\sum a_i' E(Z_i) = E(Z_o) \quad (6.12)$$

The kriging system is represented as:

$$\sum_j a_j' \gamma_{ij}' + \mu' = \gamma_{oi}' \quad (6.13)$$

(the accent denotes the inclusion of the error component)

It has been shown that the relations between terms with

and without the inclusion of errors are as follows:

$$\text{the weighted coefficients} \quad a'_i = a_i \quad (6.14)$$

$$\text{the Lagrange multiplier} \quad \mu' = \mu + \sigma_\delta^2 \quad (6.15)$$

$$\text{the variance} \quad \sigma'^2 = \sigma^2 + \sigma_\delta^2 = a_0 \sigma_\delta^2 + \sigma_\delta^2 = (1 + a_0) \sigma_\delta^2 \quad (6.16)$$

When the kriging position coincides with one of the observation points and it is done without filtering of the error, the variance is zero. When kriging with filtering of the error, all coefficients a_i will tend towards a'_i when the kriging position approaches the observation point, without experiencing any jump or discontinuity.

In general, kriging with a variogram where nugget effect occurs, there will be a sudden jump of σ_δ^2 (without filtering off the error) at the level of the observation point. This kriging variance is equal to $(1 + a_i)$ times the nugget effect, because it represents the total nugget effect of the variogram.

The experimental variogram (with error), is expressed as follows:

$$\gamma'_{ij} = \gamma_{ij} + \sigma_\delta^2 \quad (6.17)$$

From this equation, to have the error filtered off, we have to subtract the nugget effect which is equal to the variance error. This will then give the actual variogram.

Kriging with filtering can be performed with much more confidence. The variance of the residual of the estimator is much smaller when compared with classical kriging.

6.5. Global estimation.

The values of a field parameter obtained from measurements on the points of a plot, usually vary widely. In case of considering the arithmetic average of all the values, would give over- or underestimation, since the data are dependent. Against this, one can simply consider the whole plot as a block and calculate the mean value with the kriging method. The kriging variance obtained, gives the estimation variance of the mean.

Another way is to use point kriging and the point is set at infinity. This will give an estimated value equal to the mean, with the estimation variance equal to the estimation variance of the mean, plus the total variance.

6.6. Results.

To apply the technique of kriging, we used here the model variogram which was adopted to the total variogram in Chapter 4. It is a linear model, with no nugget, a slope of .0265 and a range of 142.9m.

The centre of the 14x14m plot was first kriged (table 21). A value of 0.126m/day was obtained. The observed mean of the 64 points of that study, was 0.616m/day, which is much higher than the kriged value. The kriged standard deviation is 0.91.

Four points of the 90x90m plot were also kriged (table 22). The results are compared with the respective observed values

in table 20. Each kriged point gave a value much higher than the observation. Considering logarithmic values, the confidence interval of 95% would be defined by the inequality:

$$e^{-2s} < \log K < e^{+2s} \quad (6.18)$$

where: e is the estimate ($\log \bar{K}$)

s is the estimated standard deviation, that is, the square root of the estimated variance

By removing the logarithm, we obtain:

$$\bar{K} \cdot 10^{-2s} < K < \bar{K} \cdot 10^{+2s} \quad (6.19)$$

As it can be seen in table 20, all the observations fall inside the confidence intervals of the kriged values.

Table 20. Comparison of kriged values with observations

90x90m plot point	obs. val.	kriged val.	krig. stan. dev.	lower limit	upper limit
34	0.002	0.122	1.109	0.0007	20.2
37	0.008	0.150	1.396	0.0002	92.9
64	0.001	0.140	1.175	0.0006	31.3
67	0.036	0.185	1.521	0.0002	203.8

table 21. Kriging of the middle point of the 14x14m plot

THE NOS. OF POINTS TO BE KRIGED ALONG
THE X-DIRECTON IS 1 Y-DIRECTION IS 1

THE COORD. OF THE 1ST GRID PT. IS .6663E+03, .3757E+03

THE INTERVAL BET. EACH GRID PT. IS .0, .0

THE NOS. OF MEASURED VALUES = 80

THE VARIOGRAM IS A LINEAR TYPE WITH
COEF(1)= .00000000
COEF(2)= .02652200
COEF(3)= 142.90000000

THE COMPUTATION IS WITHOUT NUGGET
FILTERING.

X-COORD.	Y-COORD.	KRIGED VALUES(LOG)	KRIGED VALUES	KRIGED VARIANCE	KRIGED STD. DEV.
666.28	375.73	-.8979681E+00	.1264829E+00	.8309892E+00	.9115861E+00

table 22. Kriging of points of the 90x90m plot

THE NOS. OF POINTS TO BE KRIGED ALONG
THE X-DIRECTION IS 1 Y-DIRECTION IS 1

THE COORD. OF THE 1ST GRID PT. IS .6706E+03. .3990E+03

THE INTERVAL BET. EACH GRID PT. IS .0. .0

THE NOS. OF MEASURED VALUES = 80

THE VARIOGRAM IS A LINEAR TYPE WITH
COEF(1)= .00000000
COEF(2)= .02652200
COEF(3)= 142.90000000

THE COMPUTATION IS WITHOUT NUGGET
FILTERING

X-COORD.	Y-COORD.	KRIGED VALUES(LOG)	KRIGED VALUES	KRIGED VARIANCE	KRIGED STD. DEV.
670.60	399.00	-.9141920E+00	.1218451E+00	.1230172E+01	.1109131E+01

THE INTERVAL BET. EACH GRID PT. IS .0. .0

THE NOS. OF MEASURED VALUES = 80

THE VARIOGRAM IS A LINEAR TYPE WITH
COEF(1)= .00000000
COEF(2)= .02652200
COEF(3)= 142.90000000

THE COMPUTATION IS WITHOUT NUGGET
FILTERING

X-COORD.	Y-COORD.	KRIGED VALUES(LOG)	KRIGED VALUES	KRIGED VARIANCE	KRIGED STD. DEV.
700.60	399.90	-.8229155E+00	.1503435E+00	.1947638E+01	.1395578E+01

THE NOS. OF POINTS TO BE KRIGED ALONG
THE X-DIRECTION IS 1 Y-DIRECTION IS 1

THE COORD. OF THE 1ST GRID PT. IS .6697E+03. .4289E+03

THE INTERVAL BET. EACH GRID PT. IS .0. .0

THE NOS. OF MEASURED VALUES = 80

THE VARIOGRAM IS A LINEAR TYPE WITH
COEF(1)= .00000000
COEF(2)= .02652200
COEF(3)= 142.90000000

THE COMPUTATION IS WITHOUT NUGGET
FILTERING

X-COORD.	Y-COORD.	KRIGED VALUES(LOG)	KRIGED VALUES	KRIGED VARIANCE	KRIGED STD. DEV.
669.70	428.90	-.8543340E+00	.1398511E+00	.1381391E+01	.1175326E+01

THE NOS. OF POINTS TO BE KRIGED ALONG
THE X-DIRECTION IS 1 Y-DIRECTION IS 1

THE COORD. OF THE 1ST GRID PT. IS .6997E+03. .4299E+03

THE INTERVAL BET. EACH GRID PT. IS .0. .0

THE NOS. OF MEASURED VALUES = 80

THE VARIOGRAM IS A LINEAR TYPE WITH
COEF(1)= .00000000
COEF(2)= .02652200
COEF(3)= 142.90000000

THE COMPUTATION IS WITHOUT NUGGET
FILTERING

X-COORD.	Y-COORD.	KRIGED VALUES(LOG)	KRIGED VALUES	KRIGED VARIANCE	KRIGED STD. DEV.
699.70	429.90	-.7322733E+00	.1852365E+00	.2314327E+01	.1521357E+01

7. CONCLUSION

In the first part of the study, the data collection, and especially in the comparison between the results of the two different methods used (constant and variable head), there appear to be measurement errors, which are hard to estimate.

The observations vary from 0.0003m/day to 488m/day. Their mean is 65.8m/day, and their variance 10^4 (m/day)².

A comparison of the map of contour lines of conductivity with an existing geological map of the region, showed no correspondance between soil types and measured conductivity values.

A statistical analysis showed that the conductivity values possibly had a possible lognormal distribution. This fact was later verified with the application of the Kolmogorov-Smirnov test, for distances higher than 100m, to avoid correlation between the observations.

In a comparison of the results of the present study with two previous studies of different area sizes (Nurul, 1984 and Tan, 1986), a clear increase of the variance with the area size was observed.

In the spatial variability analysis of the present study data set, a pure nugget effect was observed, which can be explained by the fact that variations in the data exist at a scale smaller than the sampling distances.

The variogram of the present study, differs from the variograms of the two previous ones, due to the effect of the plot size. After a proportional effect scaling, the difference with one of them is eliminated. It would be interesting to compare more data sets than available.

The total variogram of the three data sets (log of K values), was computed and modeled by a linear type model with a sill of 3.8 and a range of 143m.

The kriging technique has been applied, based on the above mentioned model. The value of conductivity in points of observation of the previous studies was estimated. The results show a tendency for overestimation of the conductivity, though there is no statistically significant difference between observations and estimations.

Finally, the geostatistical method proves to be able to provide an approach for studying the spatial variability of hydraulic conductivity, and the area size of the plot proves to play an important role in such studies.

Appendix A
Program VARIO1

Program VARIO1

1. Introduction.

The program VARIO1 is used for the computation of the statistics, and the distribution analysis with the Kolmogorov-Smirnov goodness of fit test. It is adopted from "Geostatistical ore reserve estimation" (David 1977) with several alterations. It is working with FORTRAN 77. The graph plotting is with PLOTT 83.

Its application is as follows:

1. with different grid systems (in x or y direction).
2. with different direction orientation for the variogram to be computed.
3. with different angular regularisation.
4. option for working with the logarithm of the parameter.
5. option for distribution analysis with Kolmogorov-Smirnov goodness of fit test.

2. Input data files.

There are two input data files which have to be created. The first file "DATA01" contains the necessary information of how the variogram is to be computed. The next file "DANEW" consists of a heading which is to be print in the output, and the tabulated values of the parameter. The program is written in a format which will ask for the name of the input files.

2.1. DATA01 input file.

Card No.	Column No.	Format	Symbol	Description
1	1-2	I2	ILOG	1 when working with logarithmic values 0 when working with the parameter
2	1-10	F10.5	STEP	The interval with which the variograms have to be computed
3	1-10	F10.5	BORN	The maximum value of the parameter
4	1-10	F10.5	IDEF	Notation of the parameter
5	1-10	F10.0	PHI	Direction of the computation
6	1-10	F10.0	PSI	Angular regularisation of the variogram
7	1-4	I4	NS	Total number of data
8	1-2	I2	ICU	1 for the cumulative distribution analysis 0 if not required

2.2. DANEW input file.

Card No.	Column No.	Format	Symbol	Description
1-2	1-80	2(20A4)	ICOM	The heading to be printed in the output
3-103	7-16	F10.5	Y(N)	The y-coordinate
	17-26	F10.5	X(N)	The x-coordinate
	27-36	F10.7	A(N)	The value of the parameter

N.B. The plotting file and library must be called before compilation.

3. Output files.

There are two output files. The first, RESULT1, is created only in case that the distribution analysis is required, and contains it. The second one, RESULT3, contains the values of statistical parameters of the data set, and the variogram.

C
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* VARIO3 *

THIS PROGRAM IS FOR THE CALCULATION OF
1. THE MEAN, VARIANCE, SKEWNESS AND KURTOSIS
2. THE DISTRIBUTION ANALYSIS WITH KOLMOGOROV-SMIRNOV
GOODNESS OF FIT TEST OPTION.
3. AND THE VARIOGRAMS WITH DIFFERENT INTERVAL,
DIRECTION, ANG. REGULARISATION AND LOGARITHM
OPTIONS OF A SET OF FIELD DATA.

PROGRAM VARIO1
DIMENSION ICOM(40), DIV(250), PERC(250), TD(250)
DIMENSION A(250), X(250), Y(250), Z(250), ZZ(250)
C *****
C DIMENSION DZMAX(40), IMAX(40), JMAX(40)
C *****
COMMON ST, MOY, NS, NX, ILOG
REAL MOY, M1, M2
INTEGER EFF(40), BINP, BSUP
CHARACTER DATA01*7, MADATA*7
DIMENSION DISTOT(40), S1(40), S2(40)
DIMENSION XG(42), YG(42)
WRITE(*, 1)
1 FORMAT(10X, 'GIVE THE NAME OF THE TWO DATA FILES', /,
* 10X, '(7 CHARACTERS)')
READ(*, 2) DATA01
2 FORMAT(A7)
READ(*, 2) MADATA
OPEN(1, FILE=DATA01)
OPEN(2, FILE=MADATA)
OPEN(3, FILE='RESULT1')
OPEN(4, FILE='RESULT3')

C
C
C
C

READING INPUT DATA

READ(1, 15) ILOG
15 FORMAT(I2)
READ(1, 17) STEP
17 FORMAT(F10.5)
READ(1, 17) BORN
READ(1, 30) IDEF
30 FORMAT(A4)
READ(1, 70) PHI
70 FORMAT(F10.0)
READ(1, 70) PSI
READ(1, 95) NS
95 FORMAT(I4)
READ(1, 15) ICU
READ(2, 10) ICOM
10 FORMAT(20A4, /, 20A4)
DO 160 N=1, NS
READ(2, 100) INOM, Y(N), X(N), A(N)
100 FORMAT(A6, 2F10.5, F10.7)
103 IF(ILOG.EQ.0) GO TO 160
IF(A(N).EQ.0) GO TO 160
A(N)=ALOG10(A(N))
NO=NO+1
160 CONTINUE
IF(ILOG.EQ.0) GO TO 125
BORN=ALOG10(BORN)
125 WRITE(3, 170) NS
170 FORMAT(1H, 'NUMBER OF SAMPLES= ', I4)
DO 180 LP3=1, NS
180 Z(LP3)=A(LP3)
APSI=22./7.*PSI/360.
T1=COS(APSI)
APHI=22./7.*PHI/180.
CA=COS(APHI)
SA=SIN(APHI)
DO 200 LP2=1, 40
EFF(LP2)=0
DISTOT(LP2)=0.
S1(LP2)=0.
S2(LP2)=0.
200 CONTINUE

C
C
C

COMPUTATION OF MEAN, VARIANCE, SKEWNESS AND KURTOSIS

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227 SM=SB=SC=SD=0
DO 215 LP1=1, NS
IF (Z(LP1).EQ.0) GO TO 215
NN=NN+1
ZLP=Z(LP1)
SM=SM+Z(LP1)
SB=SB+ZLP*ZLP
SC=SC+ZLP**3
SD=SD+ZLP**4
215 CONTINUE
WRITE(*,*) 'SUM=', SM
MOY=SM/FLOAT(NS)
VRNCE=SB/(NS-1)-(MOY**2)*NS/(NS-1)
ST=SQRT(VRNCE)
SKEW=(SC/FLOAT(NS)-3.*MOY*SB/FLOAT(NS)+2.*MOY**3)/ST**3
CURT=(SD/FLOAT(NS)-4.*MOY*SC/FLOAT(NS)+6.*MOY**2*SB/
*FLOAT(NS)-3.*MOY**4)/ST**4

```

C
C
C

DISTRIBUION ANALYSIS WITH K-S GOODNESS OF FIT TEST

```

IF(ICU.EQ.0) GO TO 230
NX=0
DO 25 I=1, NS
IF(Z(I).EQ.0) GO TO 25
NX=NX+1
ZZ(NX)=Z(I)
25 CONTINUE
DO 28 I=1, NX
DO 26 J=1, NX
IF(ZZ(J).GT. ZZ(I)) GO TO 37
GO TO 26
37 H=ZZ(I)
ZZ(I)=ZZ(J)
ZZ(J)=H
26 CONTINUE
28 CONTINUE
D2=0
WRITE(3, 147)
147 FORMAT(///, 3X, 'RANK', 2X, 'Z VAL', 2X, 'NOR. VAL', 2X, 'EXPT. PROB',
* 2X, 'THEO. PROB', 5X, 'D1', 6X, 'D2', 6X, 'D')
DO 106 J=1, NX
DIV(J)=(ZZ(J)-MOY)/ST
106 TD(J)=0.5*(1.+ERF(DIV(J)/SQRT(2.)))
DO 91 I=1, NX
PERC(I)=(I-0.5)/NX
DIV(I)=(ZZ(I)-MOY)/ST
TD(I)=0.5*(1.+ERF(DIV(I)/SQRT(2.)))
D1=ABS(PERC(I)-TD(I))
D=D1
IF(D2.GT.D1) D=D2
IF(D.GT.DMX) DMX=D
J=I+1
WRITE(3, 149) I, ZZ(I), DIV(I), PERC(I), TD(I), D1, D2, D
91 D2=ABS(PERC(I)-TD(J))
WRITE(3, 179) DMX
179 FORMAT(///, DMX = , F8.3)
149 FORMAT(I6, 2F8.3, 2F12.7, 3F8.3)

```

C
C
C

RESULTS OF K-S TEST FOR SIGNIF. LEVELS OF 5% AND 10%

```

NUM1=5
NUM2=10
F5=1.36/SQRT(NS+.0)
F10=1.22/SQRT(NS+.0)
IF (F5.GT.DMX) THEN
WRITE(3, 1001) NUM1, DMX, F5
ELSE
WRITE(3, 1002) NUM1, DMX, F5
WRITE(3, *)
IF (F10.GT.DMX) THEN
WRITE(3, 1001) NUM2, DMX, F10
ELSE
WRITE(3, 1002) NUM2, DMX, F10
ENDIF

```

```

1001 FORMAT(//,3X,'THE NORMALITY OF THE OBSERVATIONS IS ACCEPTED',
*,/,3X,'FOR SIGNIFICANCE LEVEL',3X,I2,'%',/,3X,'SINCE DMAX=',
*2X,F8.3,3X,'AND THE LIMIT IS',2X,F8.3)
1002 FORMAT(//,3X,'THE NORMALITY OF THE OBSERVATIONS IS REJECTED',
*,/,3X,'FOR SIGNIFICANCE LEVEL',3X,I2,'%',/,3X,'SINCE DMAX=',
*2X,F8.3,3X,'AND THE LIMIT IS',2X,F8.3)
C
WRITE(3,137)MOY,ST
137 FORMAT(///,10X,'THE MEAN IS ',F10.6,3X,'AND STD. IS ',F10.6)
CALL GROPEN
C
C
C
GRAPH PLOTTING OF THE THEORE. AND EXPERI. CURVES
CALL PLSIZE(25.,15.)
CALL BOUNDS(0.,0.,0.,0.)
CALL OPTION('TL')
CALL XLABEL('THE RANKED RESIDUALS')
CALL YLABEL('THE EXP AND THEO. DISTRIBUTIONS_')
CALL CUTYPE('DA')
DO 20 I=1,NX
X1=ZZ(I)
Y1=PERC(I)
20 CALL DRAW(X1,Y1)
CALL CUNEXT
CALL CUTYPE('SO')
DO 40 I=1,NX
X1=ZZ(I)
Y2=TD(I)
40 CALL DRAW(X1,Y2)
CALL CUNEXT
CALL CUTYPE('SO')
CALL DRAW(-2.2,1.2)
CALL DRAW(-1.7,1.2)
CALL ADDCMT(-1.5,1.2,'_THEO. DISTRIBUTION_')
CALL CUNEXT
CALL CUTYPE('DA')
CALL DRAW(-2.2,1.15)
CALL DRAW(-1.7,1.15)
CALL ADDCMT(-1.5,1.15,'_EXP. DISTRIBUTION_')
CALL CRCLOS
C
C
C
COMPUTATION OF VARIOGRAM
230 DO 290 LP1=1,NS
IF(Z(LP1).EQ.0) GO TO 290
IF(Z(LP1)-BORN) 240,290,290
240 I2=LP1+1
IF(I2.GT.NS) GO TO 290
DO 280 LP2=I2,NS
IF(Z(LP2).EQ.0) GO TO 280
IF(Z(LP2)-BORN)250,280,280
250 D2=(X(LP1)-X(LP2))**2+(Y(LP1)-Y(LP2))**2
IF(D2.LT.0.000001) GO TO 280
D1=SGRT(D2)
CC=(X(LP1)-X(LP2))*CA/D1+(Y(LP1)-Y(LP2))*SA/D1
CC1=ABS(CC)
IF(CC1.GT.T1) GO TO 260
GO TO 280
260 RR=D1/STEP
IF(RR-40.) 270,280,280
270 IC=RR+1
DELTZ=CC*(Z(LP1)-Z(LP2))/CC1
EFF(IC)=EFF(IC)+1
S1(IC)=S1(IC)+DELTZ
S2(IC)=S2(IC)+DELTZ**2
DISTOT(IC)=DISTOT(IC)+D1
C *****
IF(DZMAX(IC).GT.DELTZ) GO TO 280
IMAX(IC)=LP1
JMAX(IC)=LP2
DZMAX(IC)=DELTZ
C *****

```

```

280 CONTINUE
290 CONTINUE
300 ITEN=IDF
217 WRITE(4,310)ICOM,PSI
310 FORMAT(1H,57X,'V A R I O G R A M',///,
1      1H,2(27X,20A4/1H),40X,
1      '( WITH A FIELD OF ',F4.0,
1      ' DEGREES IN EACH DIRECTION )',/
1      1H,100X,16(1H.))
      WRITE(4,320)
320 FORMAT(101X,1H.,14X,1H.)
      WRITE(4,330)ITEN
330 FORMAT(1H,100X,1H.,5X,A4,5X,1H.)
      WRITE(4,320)
      WRITE(4,340)STEP
340 FORMAT(1H,27HSTEP IN METER = ,E10.4,
1      63X,16(1H.))
      WRITE(4,350)BORN
350 FORMAT(1H,27HUPPER LIMIT FOR Z = ,E10.4,63X,14(1H.))
      WRITE(4,360)
360 FORMAT(1H,100X,1H.,12X,1H.)
      WRITE(4,370)MOY,PHI
370 FORMAT(1H,'GENERAL MEAN OF Z = ',E10.4,62X,
1      1H.,2X,F4.0,6X,1H.)
      WRITE(4,360)
      WRITE(4,380)VRNCE
380 FORMAT(1H,'GENERAL VARIANCE OF Z = ',E10.4,62X,14(1H.))
      WRITE(4,382)SKEW
382 FORMAT(1X,'GENERAL SKEWNESS OF Z = ',E10.4,/)
      WRITE(4,384)CURT
384 FORMAT(1X,'GENERAL KURTOSIS OF Z = ',E10.4,/)
      WRITE(4,390)
390 FORMAT(1H,12X,'DISTANCE IN METER NO. OF PAIRS DRIFT
1      VARIOGRAM AVERAGE DISTANCE MAXVAR PAIR',//)
      IPT=0
      DO 430 LP2=1,40
      IF(EFF(LP2)) 430,430,420
420 M1=S1(LP2)/FLOAT(EFF(LP2))
      M2=0.5*S2(LP2)/FLOAT(EFF(LP2))
      DISMOY=DISTOT(LP2)/FLOAT(EFF(LP2))
      BINF=STEP*LP2-STEP
      BSUP=STEP*LP2
      IPT=IPT+1
      XG(IPT)=DISMOY
      YG(IPT)=M2
      WRITE(4,425)BINF,BSUP,EFF(LP2),M1,M2,DISMOY,IMAX(IC),JMAX(IC)
      IC=IC+1
425 FORMAT(1X,12X,I4,5H ----,I4,8X,I8,7X,E10.3,4X,E13.4,14X,F6.1
*,10X,I2,'---',I3)
430 CONTINUE
      WRITE(*,*)IPT
      WRITE(4,9)
      9 FORMAT(3X,'99.99')
      END

```

20. 11. 37. UCLP, 5B, DEFTERM, 0.320KLNS.

Appendix B
Program KRIG01

Program KRIG01

1. Introduction.

The program KRIG01 is used for kriging. The main output is the kriging estimator. It is adopted from "Geostatistical ore reserve estimation" (David 1977) with some alterations. It works on FORTRAN 77.

It can be applied as follows:

1. The number of kriged points can vary.
2. The kriging position can be chosen as desired.
3. It includes the option of working with logarithmic values.
4. It includes the option of filtering out the nugget effect.
5. There is the possibility of choosing different type of variogram models.

2. Input data files.

Two input data files are required. One (DANEW, as used in program VARIO1), contains the observed values of the parameter. The other one, DATA07, should contain the information required for kriging. The contents of this file should be as follows:

Card No.	Col. No.	Format	Symbol	Description
1	1-2	I2	NX	The number of points to
2	1-2	I2	NY	be kriged along the x and y direction.
3	1-10	F10.0	XC	The kriged position's

4	1-10	F10.0	YC	coordinates
5	1-10	F10.0	PASX	Interval between each
6	1-10	F10.0	PASY	kriged point in the x and y direction.
7	1-3	I3	NP	Number of measured points
8	1-3	I3	NT	1 for working with logar. values, 0 when not req.
9	1-3	I3	NU	1 for nugget effect filtering, 0 when it is not required.
10	1-3	I3	NG	Type of variogram model to be used: 1 - for spherical 2 - for linear 3 - for power type 4 - for logarithmic 5 - for exponential 6 - for Gaussian
11	1-10	F10.0	COEF1	The nugget effect
12	1-10	F10.0	COEF2	Value of the 2nd coef.
13	1-10	F10.0	COEF3	Value of the 3rd coef.

PROGRAM KRJGEO1

THIS IS A POINT KRIGING PROGRAMME
WITH NUGGET FILTERING OPTION
FOR A GRID NX BY NY.
LABORATORY OF HYDROLOGY, VUB, 1986.
DIMENSION X(120), Y(120), T(120)
DIMENSION ICOM(40)
DIMENSION A(120, 120), B(120), GAMM(120), COEF(3)
DIMENSION XF(16), YF(16)
COMMON NG, COEF, NU
OPEN(1, FILE='DATA07')
OPEN(2, FILE='DAMEW2')
OPEN(3, FILE='RESULTS')

READING INPUT DATA

```
24 READ(1,24)NX
   FORMAT(I2)
   READ(1,24)NY
28 READ(1,28)XC
   FORMAT(F10.0)
   READ(1,28)YC
   READ(1,28)PASX
   READ(1,28)PASY
   READ(1,36)NP
36 FORMAT(I3)
   READ(1,42)NT
42 FORMAT(I1)
   READ(1,42)NU
   READ(1,42)NG
29 READ(1,29)COEF(1)
   FORMAT(F10.0)
   READ(1,29)COEF(2)
   READ(1,29)COEF(3)
52 WRITE(3,52)NX,NY
   FORMAT(10X, 'THE NOS. OF POINTS TO BE KRIGED ALONG',/,
   $      10X, 'THE X-DIRECTON IS ', I2, 2X, 'Y-DIRECTION IS ', I2, 2X, //)
54 WRITE(3,54)XC, YC
   FORMAT(10X, 'THE COORD. OF THE 1ST GRID PT. IS ', E13.4, ', ',
   $      E13.4, //)
56 WRITE(3,56)PASX, PASY
   FORMAT(10X, 'THE INTERVAL BET. EACH GRID PT. IS ', F5.1,
   $      ', ', F5.1, //)
58 WRITE(3,58)NP
   FORMAT(10X, 'THE NOS. OF MEASURED VALUES =', I3, //)
   IF(NG.EQ.1) THEN
62 WRITE(3,62)COEF(1), COEF(2), COEF(3)
   FORMAT(10X, 'THE VARIOGRAM IS A SPHERICAL TYPE WITH',/,
   $      10X, 'COEF(1)=' , F18.8, /, 10X, 'COEF(2)=' , F18.8, /,
   $      10X, 'COEF(3)=' , F18.8, //)
   ELSE IF(NG.EQ.2) THEN
64 WRITE(3,64)COEF(1), COEF(2), COEF(3)
   FORMAT(10X, 'THE VARIOGRAM IS A LINEAR TYPE WITH',/,
   $      10X, 'COEF(1)=' , F18.8, /, 10X, 'COEF(2)=' , F18.8, /,
   $      10X, 'COEF(3)=' , F18.8, //)
   ELSE IF(NG.EQ.3) THEN
66 WRITE(3,66)COEF(1), COEF(2), COEF(3)
   FORMAT(10X, 'THE VARIOGRAM IS A POWER TYPE WITH',/,
   $      10X, 'COEF(1)=' , F18.8, /, 10X, 'COEF(2)=' , F18.8, /,
   $      10X, 'COEF(3)=' , F18.8, //)
   ELSE IF(NG.EQ.4) THEN
68 WRITE(3,68)COEF(1), COEF(2), COEF(3)
   FORMAT(10X, 'THE VARIOGRAM IS A LOG. TYPE WITH',/,
   $      10X, 'COEF(1)=' , F18.8, /, 10X, 'COEF(2)=' , F18.8, /,
   $      10X, 'COEF(3)=' , F18.8, //)
   ELSE IF(NG.EQ.5) THEN
70 WRITE(3,70)COEF(1), COEF(2), COEF(3)
   FORMAT(10X, 'THE VARIOGRAM IS A EXPO. TYPE WITH',/,
   $      10X, 'COEF(1)=' , F18.8, /, 10X, 'COEF(2)=' , F18.8, /,
   $      10X, 'COEF(3)=' , F18.8, //)
   ELSE IF(NG.EQ.6) THEN
72 WRITE(3,72)COEF(1), COEF(2), COEF(3)
   FORMAT(10X, 'THE VARIOGRAM IS A GAUSSIAN TYPE WITH',
   $      /, 10X, 'COEF(1)=' , F18.8, /, 10X, 'COEF(2)=' ,
   $      F18.8, /, 10X, 'COEF(3)=' , F18.8, //)
   ENDIF
   IF(NU.EQ.1) THEN
74 WRITE(3,74)
   FORMAT(10X, 'THE COMPUTATION IS WITH THE NUGGET',/,
   $      10X, 'FILTERING. ', //)
   ELSE
76 WRITE(3,76)
   FORMAT(10X, 'THE COMPUTATION IS WITHOUT NUGGET',/,
   $      10X, 'FILTERING. ', //)
   ENDIF
```

```

      READ(2,78) ICOM
78  FORMAT(20(A4),/,20(A4))
      DO 102 I=1,NP
      READ (2,14)X(I),Y(I),T(I)
14  FORMAT(5X,2F10.5,F10.7)
C
C      END OF READING INPUT FILES
C
      IF(NT.EQ.0) GO TO 102
      IF(T(I).EQ.0) GO TO 102
      T(I)=ALOG10(T(I))
C 102 CONTINUE
C      WRITE(3,8)
      8  FORMAT(19X,'Y-COORD',8X,'X-COORD',6X,'VALUES')
      DO 120 I=1,NP
C      WRITE(3,12)X(I),Y(I),T(I)
C 12  FORMAT(10X,2(5X,F10.2),5X,E10.3)
120 CONTINUE
      SIG27=0.
      NN=NP+1
      DO 300 L=1,NP
      DO 300 K=1,L
          ZZ=0
      CALL GAMMA (X(L),Y(L),X(K),Y(K),G,ZZ)
300  A(L,K)=A(K,L)=G
      DO 400 L=1,NP
400  A(NN,L)=A(L,NN)=1.
      A(NN,NN)=0.
      CALL MATINV(A,NN,DET)
      IF(NT.EQ.1)GO TO 420
      WRITE(3,450)
450  FORMAT(//////,9X,'X-COORD.',3X,'Y-COORD.',3X,'KRIGED',
$      12X,'KRIGED',8X,'KRIGED',/,31X,'VALUES',
$      11X,'VARIANCE',8X,'STD. DEV. ')
      GO TO 410
420  WRITE(3,430)
430  FORMAT(//////,9X,'X-COORD.',3X,'Y-COORD.',5X,'KRIGED',
$      3(10X,'KRIGED'),/,31X,'VALUES(LOG)',7X,'VALUES',
$      9X,'VARIANCE',8X,'STD. DEV. ')
410  DO 200 LP1=1,NX
      DO 200 LP2=1,NY
      XCEN=XC+(LP1-1)*PASX
      YCEN=YC+(LP2-1)*PASY
      DO 500 L=1,NP
      ZZ=1
      CALL GAMMA(X(L),Y(L),XCEN,YCEN,G,ZZ)
      B(L)=G
500  CONTINUE
      B(NN)=1.
      DO 510 L=1,NN
      GAMM(L)=0.
      DO 502 K=1,NN
      GAMM(L)=A(L,K)*B(K)+GAMM(L)
502  CONTINUE
      SGAMM=SGAMM+GAMM(L)
510  CONTINUE
      SOMX=0.
      SOML=0.
      DO 600 L=1,NP
600  SOMX=SOMX+T(L)*GAMM(L)
      SK=0.
      DO 610 L=1,NN
610  SK=SK+GAMM(L)*B(L)
      IF (NU.EQ.1) SK=SK+COEF(1)
      IF(SK.LE.0) SK=0
      SKRT=SQRT(SK)
      IF(NT.EQ.1) GO TO 630
      WRITE(3,700) XCEN,YCEN,SOMX,SK,SKRT
700  FORMAT(5X,2F10.2,3E16.7)
      GO TO 200
630  ASOMX=10**SOMX
      WRITE(3,720)XCEN,YCEN,SOMX,ASOMX,SK,SKRT
720  FORMAT(5X,2F10.2,4E16.7)
200  CONTINUE
      STOP
      END

```



```

SUBROUTINE GAMMA(X1, Y1, X2, Y2, GAMMB, ZZ)
DIMENSION COEF(3)
COMMON NG, COEF, NU
D=SQRT((X1-X2)**2+(Y1-Y2)**2)
IF(NG.EQ.1) THEN
  IF(D.GT.COEF(3)) THEN
    GAMMB=COEF(1)+0.5*COEF(2)*(3*D/COEF(3)-(D/COEF(3)**3))
  ELSE
    GAMMB=COEF(1)+COEF(2)
  ENDIF
ELSE IF(NG.EQ.2) THEN
  IF(D.GT.COEF(3)) THEN
    GAMMB=COEF(1)+COEF(2)*COEF(3)
  ELSE
    GAMMB=COEF(1)+COEF(2)*D
  ENDIF
ELSE IF(NG.EQ.3) THEN
  GAMMB=COEF(1)+COEF(2)*D**COEF(3)
ELSE IF(NG.EQ.4) THEN
  GAMMB=COEF(1)+COEF(2)*ALOG(1+COEF(3)*D)
ELSE IF(NG.EQ.5) THEN
  GAMMB=COEF(1)+COEF(2)*(1-EXP(-(COEF(3)*D)))
ELSE IF(NG.EQ.6) THEN
  GAMMB=COEF(1)+COEF(2)*(1-EXP(-(COEF(3)*D**2)))
ENDIF
IF(ZZ.EQ.1) THEN
  IF(NU.EQ.1) GAMMB=GAMMB-COEF(1)
ENDIF
IF(D.LE.0.001) GAMMB=0.0
RETURN
END

```

C

```

SUBROUTINE MATINV (ARRAY, NORDER, DET)
DIMENSION ARRAY(120, 120), IK(120), JK(120)
10 DET = 1.
11 DO 100 K=1, NORDER

```

C
C
C

FIND LARGEST ELEMENT ARRAY(I, J) IN REST OF MATRIX

```

  AMAX = 0.
21 DO 30 I=K, NORDER
  DO 30 J=K, NORDER
23 IF(ABS(AMAX)-ABS(ARRAY(I, J))) 24, 24, 30
24 AMAX = ARRAY(I, J)
  IK(K) = I
  JK(K) = J
30 CONTINUE

```

C
C
C

INTERCHANGE ROWS AND COLUMNS TO PUT AMAX IN ARRAY(K, K)

```

31 IF(AMAX) 41, 32, 41
32 DET = 0.
  GOTO 140
41 I = IK(K)
  IF(I=K) 21, 51, 43
43 DO 50 J=1, NORDER
  SAVE = ARRAY(K, J)
  ARRAY(K, J) = ARRAY(I, J)
50 ARRAY(I, J) = -SAVE
51 J = JK(K)
  IF(J=K) 21, 61, 53
53 DO 60 I=1, NORDER
  SAVE = ARRAY(I, K)
  ARRAY(I, K) = ARRAY(I, J)
60 ARRAY(I, J) = -SAVE

```

C

```

C
C
C   ACCUMULATE ELEMENTS OF INVERSE MATRIX
61 DO 70 I=1,NORDER
   IF(I-K) 63,70,63
63 ARRAY(I,K) = -ARRAY(I,K)/AMAX
70 CONTINUE
71 DO 80 I=1,NORDER
   DO 80 J=1,NORDER
   IF(I-K) 74,80,74
74 IF(J-K) 75,80,75
75 ARRAY(I,J) = ARRAY(I,J)+ARRAY(I,K)*ARRAY(K,J)
80 CONTINUE
81 DO 90 J=1,NORDER
   IF(J-K) 83,90,83
83 ARRAY(K,J) = ARRAY(K,J)/AMAX
90 CONTINUE
   ARRAY(K,K) = 1./AMAX
100 DET = DET*AMAX
C
C
C   RESTORE ORDERRING OF MATRIX
101 DO 130 L=1,NORDER
   K = NORDER-L+1
   J = IK(K)
   IF(J-K) 111,111,105
105 DO 110 I=1,NORDER
   SAVE = ARRAY(I,K)
   ARRAY(I,K) = -ARRAY(I,J)
110 ARRAY(I,J) = SAVE
111 I = JK(K)
   IF(I-K) 130,130,113
113 DO 120 J=1,NORDER
   SAVE = ARRAY(K,J)
   ARRAY(K,J) = -ARRAY(I,J)
120 ARRAY(I,J) = SAVE
130 CONTINUE
140 RETURN
   END

```

12. 47. 22. UCLP, 5B, DEFTERM, 0. 256KLNS.

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